## EUCLIDEAN GEOMETRY

### SOLVING GEOMETRICAL PROBLEMS

The following exercises involve the use of all the theorems established thus far.

The following strategy can be used when solving riders:

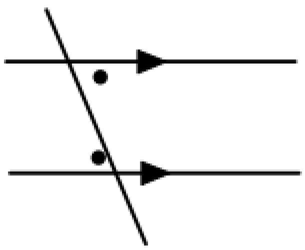
**STEP 1:** Analyse the RTP (required to prove) in terms of **angles.**

**STEP 2:** Pay attention to the **keywords** given. Look for information in the diagram which might prove useful. **Use colours** to mark off equal angles / sides.

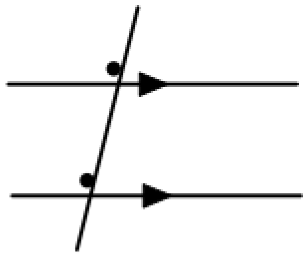
**STEP 3:** Brainstorm and develop a **rough proof.**

**STEP 4:** Rewrite a **formal proof.**

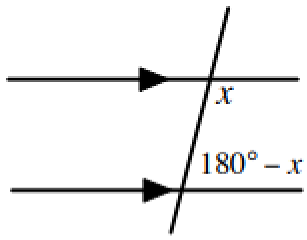
**1. When parallel lines are given**



alt $\angle s$ =

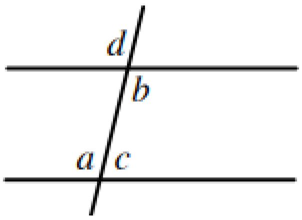


corr $\angle s$ =




co-int $\angle s$ suppl

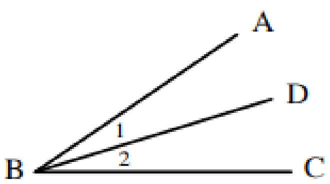
**2. How to prove that lines are parallel**



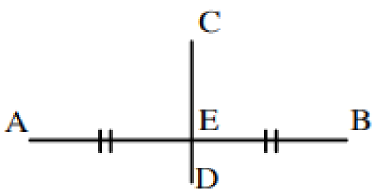

Prove that $a = b$ or $a = d$ or $b + c = 180°$

**3. Angle or line bisectors**

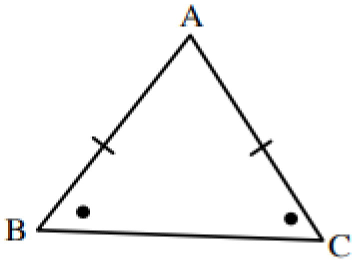
If BD bisects $A\hat{B}C$ then $\hat{B}_1 = \hat{B}_2$



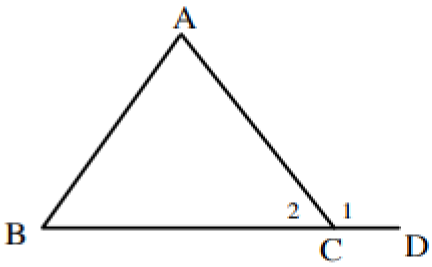

If CD bisects AB then AE = EB



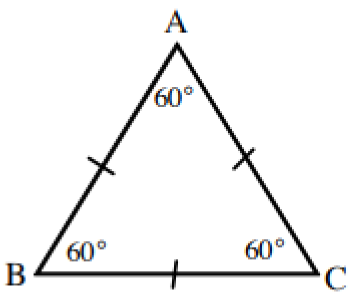
## 4. **<u>Triangle information</u>**




If $\hat{B} = \hat{C}$, then AB = AC.
If AB = AC, then $\hat{B} = \hat{C}$.
$\Delta$ABC is **<u>isosceles</u>**

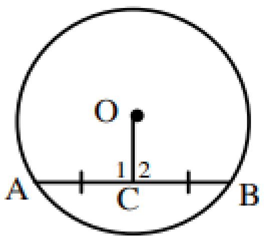



$\hat{A} + \hat{B} + \hat{C}_2 = 180°$ (sum $\angle s$ of $\Delta$)
$\hat{C}_1 = \hat{A} + \hat{B}$ (Ext $\angle$ of $\Delta$)

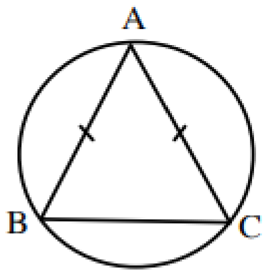



If AB = AC = BC, then $\hat{A} = \hat{B} = \hat{C} = 60°$
$\Delta$ABC is **<u>equilateral</u>**

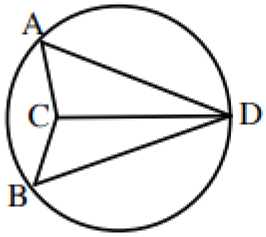
## 5. **<u>When you must prove two sides are equal</u>**




or




or

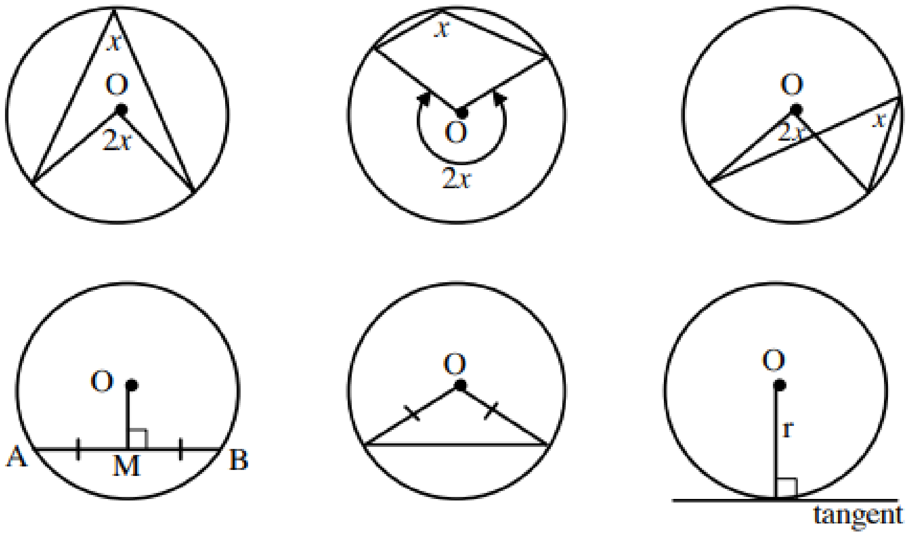



To prove AC = CB, prove $\hat{C}_1 = 90°$

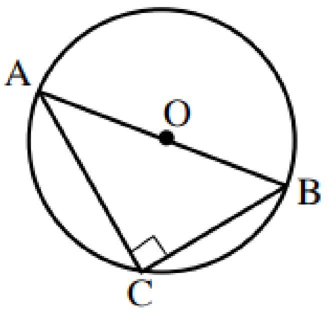
To prove AB = AC, prove $\hat{B} = \hat{C}$

To prove AD = BD, try prove $\Delta ACD \equiv \Delta BCD$

**6.** **Centre of a circle given**

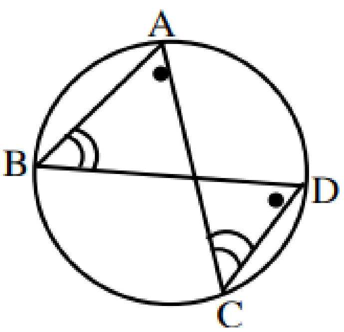



**7.** **Diameter given**



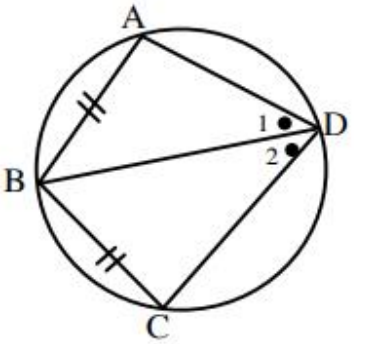

If AOB is diameter then $\hat{C} = 90°$
($\angle$ in semi circle)

**8.** **Angles in the same segment given (NB: angles formed at the circumference)**

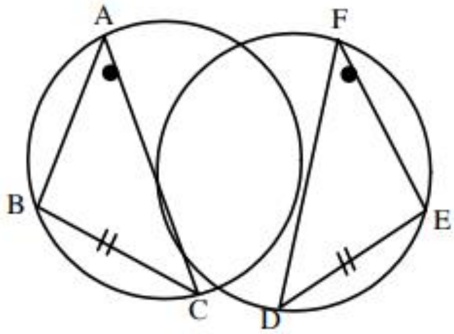



If A, B, C and D are concyclic (lie on a circle) and if AB, AC, BD and CD are chords of the circle, then $\hat{A} = \hat{D}$ and $\hat{B} = \hat{C}$
($\angle$ in same segment) or
(line/arc subtends equal angles)

## 9. Chords in a circle

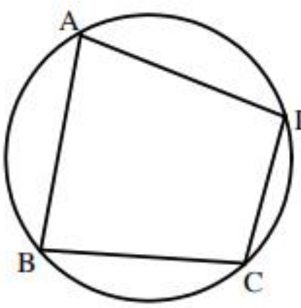


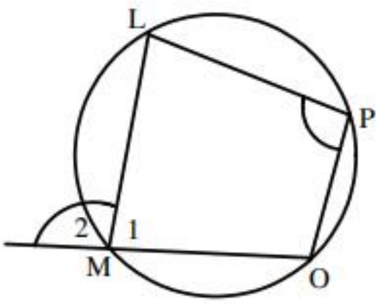

If AB = BC, then $\hat{D}_1 = \hat{D}_2$

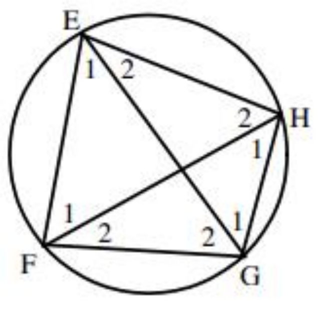



If ABC and DEF are equal circles.
then $\hat{A} = \hat{F}$ if BC = DE

## 10. Cyclic quadrilateral given



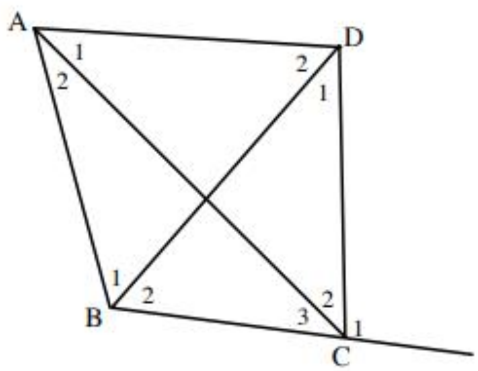

If ABCD is cyclic then
$\hat{A} + \hat{C} = 180°$ and
$\hat{B} + \hat{D} = 180°$
(opp $\angle s$ of cyclic quad)

If LMOP is cyclic
then $\hat{M}_2 = \hat{P}$
(Ext $\angle$ cyclic quad)

If EFGH is cyclic then
$\hat{E}_1 = \hat{H}_1$, $\hat{E}_2 = \hat{F}_2$,
$\hat{H}_2 = \hat{G}_2$, $\hat{G}_1 = \hat{F}_1$
($\angle s$ in same segment)

## 11. How to prove that a quadrilateral is cyclic

ABCD would be a cyclic quadrilateral if you could prove **one** of the following

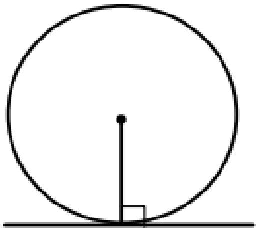



**Condition 1:**
$\left(\hat{A}_1 + \hat{A}_2\right) + \left(\hat{C}_2 + C_3\right) = 180°$ or
$\left(\hat{B}_1 + \hat{B}_2\right) + \left(\hat{D}_1 + \hat{D}_2\right) = 180°$

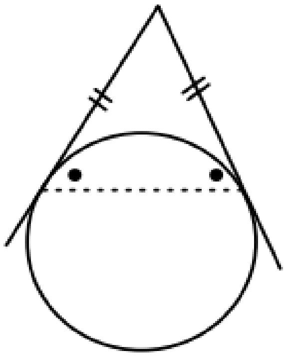
**Condition 2:**
$\hat{C}_1 = \hat{A}_1 + \hat{A}_2$

**Condition 3:**
$\hat{A}_1 = \hat{B}_2$ or $\hat{A}_2 = \hat{D}_1$ or $\hat{B}_1 = \hat{C}_2$ or $\hat{D}_2 = \hat{C}_3$

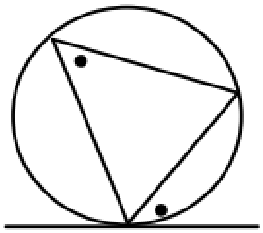
## 12. Tangents to circles given



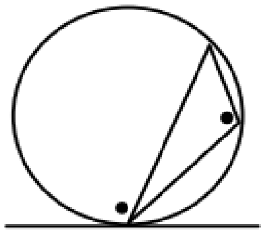
tan ⊥ rad



Tangents from the same point

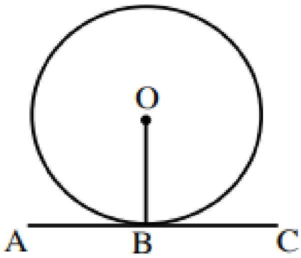


Tan-chord
(acute case)

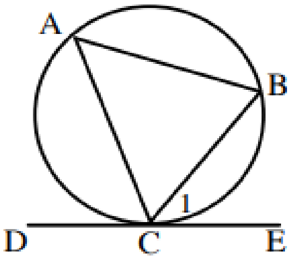


Tan-chord
(obtuse case)

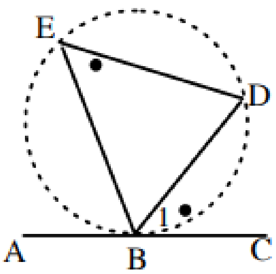
## 13. How to prove that a line is a tangent to a circle




ABC is a tangent if $O\hat{B}C = 90°$




DCE is a tangent if $\hat{C}_1 = \hat{A}$




ABC would be a tangent to the “imaginary” circle drawn through EBD if $\hat{B}_1 = \hat{E}$